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OCR A Level Computer Science

H446 – Paper 1

2

Simplifying Boolean expressions

Unit 8

Boolean Algebra

PG ONLINE

Objectives

- Use the following rules to derive or simplify statements in Boolean algebra:
  - de Morgan’s Laws
  - distribution
  - association
  - commutation
  - double negation
- Write a Boolean expression for a given logic gate circuit, and vice versa

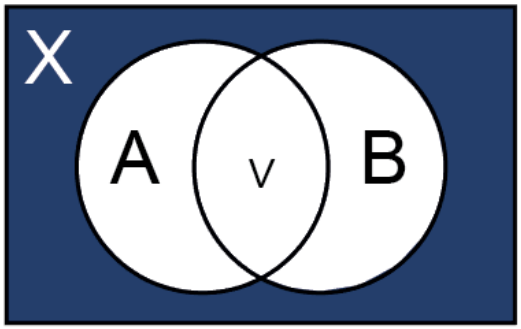
Augustus de Morgan (1806 - 71)

- de Morgan was a mathematician and logician who formulated laws to simplify Boolean expressions
- This had little practical use in his lifetime but became of major significance with the advent of computers

De Morgan's first law

- This states that

$$\neg(A \vee B) = \neg A \wedge \neg B$$



- Looking at the Venn diagram, the white area represents A OR B i.e $(A \vee B)$
- X represents all the blue area, NOT (A OR B) i.e. $\neg(A \vee B)$
- The blue area is everything that is (NOT A) AND (NOT B) i.e. $\neg A \wedge \neg B$

De Morgan's first law

- Complete the truth table to show that de Morgan's Law is true, i.e. **$\neg(A \vee B) = \neg A \wedge \neg B$**

A	B	$A \vee B$	$\neg(A \wedge B)$
0	0		
0	1		
1	0		
1	1		

$\neg A \wedge \neg B$	$\neg B$	$\neg A$
		1
		1
		0
		0

De Morgan's first law

- Complete the truth table to show that de Morgan's Law is true, i.e. **$\neg(A \vee B) = \neg A \wedge \neg B$**

A	B	$A \vee B$	$\neg(A \wedge B)$
0	0	0	
0	1	1	
1	0	1	
1	1	1	

$\neg A \wedge \neg B$	$\neg B$	$\neg A$
		1
		1
		0
		0

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- Complete the truth table to show that de Morgan's Law is true, i.e. **$\neg(A \vee B) = \neg A \wedge \neg B$**

A	B	$A \vee B$	$\neg(A \wedge B)$
0	0	0	**1**
0	1	1	**0**
1	0	1	**0**
1	1	1	**0**

$\neg A \wedge \neg B$	$\neg B$	$\neg A$
		1
		1
		0
		0

De Morgan's first law

- Complete the truth table to show that de Morgan's Law is true, i.e. **¬(A ˅ B) = ¬A ˄ ¬B**

A	B	A ˅ B	¬(A ˄ B)
0	0	0	**1**
0	1	1	**0**
1	0	1	**0**
1	1	1	**0**

¬A ˄ ¬B	¬B	¬A
	1	1
	0	1
	1	0
	0	0

De Morgan's first law

- Complete the truth table to show that de Morgan's Law is true, i.e. **¬(A ˅ B) = ¬A ˄ ¬B**

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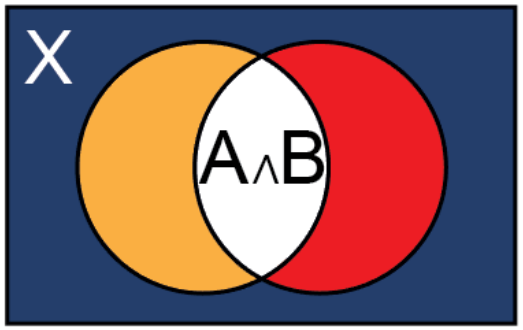
0	0	0	**1**
0	1	1	**0**
1	0	1	**0**
1	1	1	**0**

¬A ˄ ¬B	¬B	¬A
1	1	1
0	0	1
0	1	0
0	0	0

De Morgan's second law

- This states that



- Looking at the Venn diagram, if **X =** ,
  X cannot be in the white area, so it must be in the red, orange or blue area
- That is, X is either not in A, or not in B, or not in either
- This is the definition of **X =**

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

A	B	A ^ B	¬(A ^ B)
0	0		
0	1		
1	0		
1	1		

¬A ˅ ¬B	¬B	¬A
		1
		1
		0
		0

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

A	B	A ^ B	¬(A ^ B)
0	0	0	
0	1	0	
1	0	0	
1	1	1	

¬A ˅ ¬B	¬B	¬A
		1
		1
		0
		0

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

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0	0	0	**1**
0	1	0	**1**
1	0	0	**1**
1	1	1	**0**

¬A ˅ ¬B	¬B	¬A
		1
		1
		0
		0

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

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0	0	0	**1**
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1	0	0	**1**
1	1	1	**0**

¬A ˅ ¬B	¬B	¬A
	1	1
	0	1
	1	0
	0	0

De Morgan's second law

- Complete the truth table to show that de Morgan's Law is true, i.e.

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0	0	0	**1**
0	1	0	**1**
1	0	0	**1**
1	1	1	**0**

¬A ˅ ¬B	¬B	¬A
1	1	1
1	0	1
1	1	0
0	0	0

Implementing de Morgan's laws

1. Invert both terms in the expression e.g. ¬P becomes P, ¬Q becomes Q
2. Change ∧ to ∨ and ∨ to ∧ (AND to OR and OR to AND)
3. Invert the result

So ¬P ∨ ¬Q = ?

¬P ∧ ¬Q = ?

Implementing de Morgan's laws

1. Invert both terms in the expression e.g. ¬P becomes P, ¬Q becomes Q
2. Change ^ to ˅ and ˅ to ^ (AND to OR and OR to AND)
3. Invert the result

So ¬P ˅ ¬Q = ¬(P ^ Q)

¬P ^ ¬Q =

Implementing de Morgan's laws

1. Invert both terms in the expression e.g. ¬P becomes P, ¬Q becomes Q
2. Change ∧ to ∨ and ∨ to ∧ (AND to OR and OR to AND)
3. Invert the result

So ¬P ∨ ¬Q = ¬(P ∧ Q)

¬P ∧ ¬Q = ¬(P ∨ Q)

Simplifying expressions

- You need to be able to simplify given Boolean expressions
- As well as de Morgan's laws, there are several other "simplification rules" that will help you to do this
- See if you can fill in right hand side of the equalities on the next slide – remember, X and Y can only be either TRUE or FALSE

1 = TRUE, 0 = FALSE

Simplifying expressions

Can you figure out the following general rules?

1. X ^ 0 =
2. X ^ 1 =
3. X ^ X =
4. X ¬X =
5. X ˅ 0 =
6. X ˅ 1 =
7. X ˅ X =
8. X ˅ ¬X =
9. ¬X =

Simplifying expressions

Nine useful rules:

1. X ^ 0 = 0
2. X ^ 1 =
3. X ^ X =
4. X ¬X =
5. X ˅ 0 =
6. X ˅ 1 =
7. X ˅ X =
8. X ˅ ¬X =
9. ¬X =

Simplifying expressions

Nine useful rules:

1. X ^ 0 = 0
2. X ^ 1 = X
3. X ^ X =
4. X ¬X =
5. X ˅ 0 =
6. X ˅ 1 =
7. X ˅ X =
8. X ˅ ¬X =
9. ¬X =

Simplifying expressions

Nine useful rules:

1. X ^ 0 = 0
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4. X ¬X =
5. X ˅ 0 =
6. X ˅ 1 =
7. X ˅ X =
8. X ˅ ¬X =
9. ¬X =

Simplifying expressions

Nine useful rules:

1. $X \wedge 0 = 0$
2. $X \wedge 1 = X$
3. $X \wedge X = X$
4. $X \ \neg X = 0$
5. $X \vee 0 =$
6. $X \vee 1 =$
7. $X \vee X =$
8. $X \vee \neg X =$
9. $\neg X =$

Simplifying expressions

Nine useful rules:

1. $X \wedge 0 = 0$
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3. $X \wedge X = X$
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Simplifying expressions

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Simplifying expressions

Nine useful rules:

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4. $X \ \neg X = 0$
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9. $\neg X =$

Simplifying expressions

Nine useful rules:

1. $X \wedge 0 = 0$
2. $X \wedge 1 = X$
3. $X \wedge X = X$
4. $X \neg X = 0$
5. $X \vee 0 = X$
6. $X \vee 1 = 1$
7. $X \vee X = X$
8. $X \vee \neg X = 1$
9. $\neg X = X$

Some more rules

Commutative rule

10. $X \wedge Y = Y \wedge X$
11. $X \vee Y = Y \vee X$

Associative rule

12. $X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$
13. $X \vee (Y \vee Z) = (X \vee Y) \vee Z$

Distributive rule

14. $X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$
15. $(X \vee Y) \wedge (W \vee Z) = (X \wedge W) \vee (X \wedge Z) \vee (Y \wedge W) \vee (Y \wedge Z)$

Absorption rules

- There are two more rules that you will find useful for simplifying Boolean expressions:

$$X \vee (X \wedge Y) = X$$

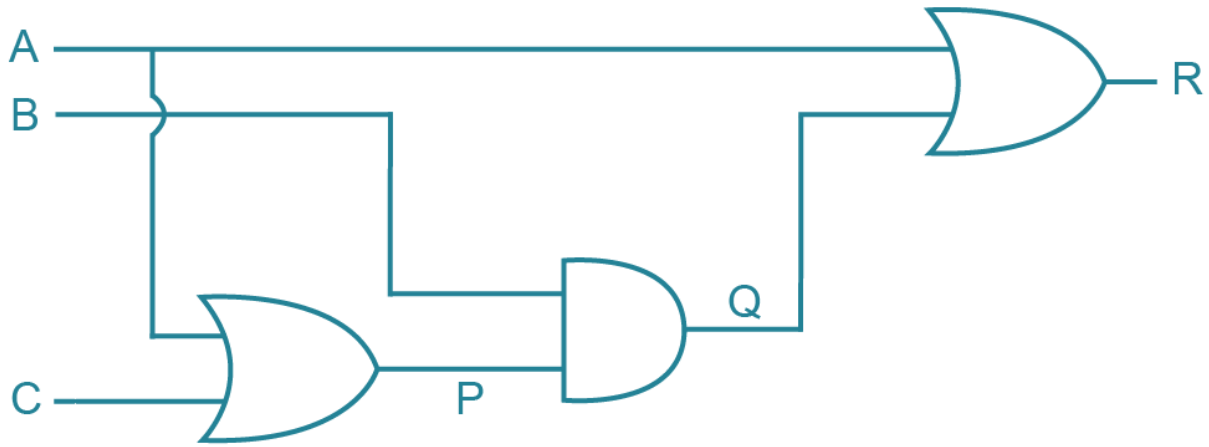
$$X \wedge (X \vee Y) = X$$

Worksheet 2

- Now try the questions in **Task 1** on **Worksheet 2**

Writing expressions representing logic circuits

- Given a logic circuit, we can break it down and find the Boolean expression that it represents
- Label the output from each gate



Writing expressions representing logic circuits

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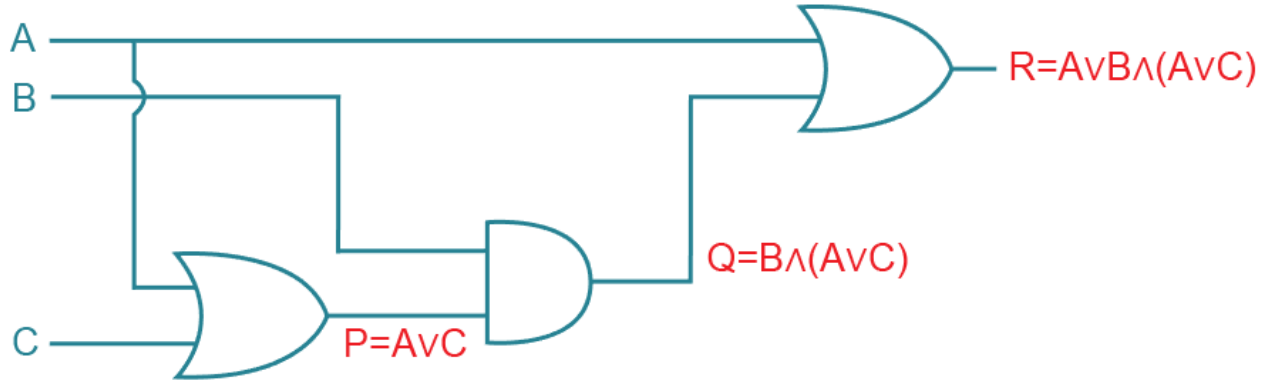


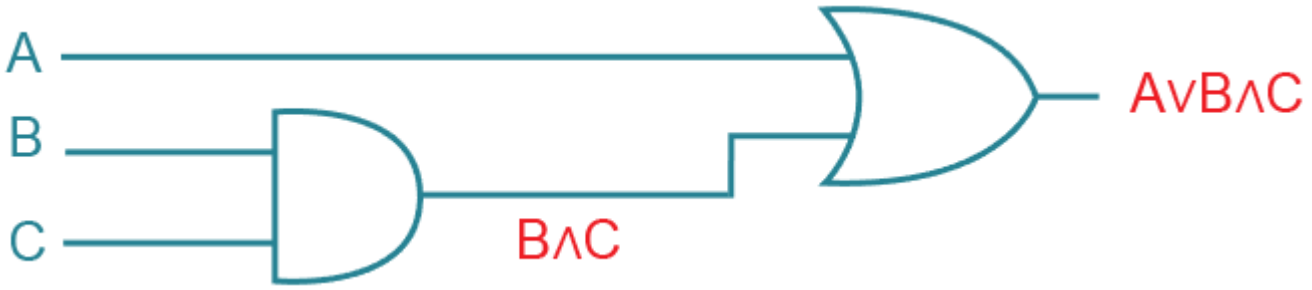
- Can you simplify R? How many gates are needed?

A simpler circuit

R = A ˅ B (A ˅ C)

= A ˅ (B A) ˅ (BC) (Distributive rule)

= A ˅ (BC) (Absorption rule)



Worksheet 6

- Now try the questions in **Task 2**
- Here's an extra one to start you off:
  - Can you simplify this circuit?

Plenary

- Boolean algebra seems tricky at first
- It’s all a matter of lots and lots of practice
- Try and familiarise yourself with de Morgan’s Laws and the other useful rules for simplifying expressions

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